Eksempel 162:
Engelsk tekst om Fermats sidste sætning

Pierre de Fermat died in 1665. Today we think of Fermat as a number theorist, in fact as perhaps the most famous number theorist who ever lived. It is therefore surprising to find that Fermat was in fact a lawyer and only an amateur mathematician. Also surprising is the fact that he published only one mathematical paper in his life, and that was an anonymous article written as an appendix to a colleague's book.

Because Fermat refused to publish his work, his friends feared that it would soon be forgotten unless something was done about it. His son, Samuel, undertook the task of collecting Fermat's letters and other mathematical papers, comments written in books, etc. with the object of publishing his father's mathematical ideas. In this way the famous »Last theorem« came to be published. It was found by Samuel written as a marginal note in his father's copy of Diophantus's *Arithmetica*.
Fermat's Last Theorem states that $x^n + y^n = z^n$ has no non-zero integer solutions for x, y and z when $n > 2$. Fermat wrote *I have discovered a truly remarkable proof which this margin is too small to contain.*
Fermat almost certainly wrote the marginal note around 1630, when he first studied Diophantus's *Arithmetica*. It may well be that Fermat realised that his *remarkable proof* was wrong, however, since all his other theorems were stated and restated in challenge problems that Fermat sent to other mathematicians. Although the special cases of $n = 3$ and $n = 4$ were issued as challenges (and Fermat did know how to prove these) the general theorem was never mentioned again by Fermat.
In fact in all the mathematical work left by Fermat there is only one proof. Fermat proves that *the area of a right triangle cannot be a square.* Clearly this means that a rational triangle cannot be a rational square. In symbols, there do not exist integers x, y, z with $x^2 + y^2 = z^2$ such that $xy/2$ is a square. From this it is easy to deduce the $n = 4$ case of Fermat's theorem.
It is worth noting that at this stage it remained to prove Fermat's Last Theorem for odd primes n only. For if there were integers x, y, z with $x^n + y^n = z^n$ then if $n = pq$, $(x^q)^p + (y^q)^p = (z^q)^p$.
Euler wrote to Goldbach on 4 August 1753 claiming he had a proof of Fermat's Theorem when $n = 3$. However his proof in *Algebra* (1770) contains a fallacy and it is far from easy to give an alternative proof of the statement which has the fallacious proof. There is an indirect way of mending the whole proof using arguments which appear in other proofs of Euler so perhaps it is not too unreasonable to attribute the $n = 3$ case to Euler.
Euler's mistake is an interesting one, one which was to have a bearing on later developments. He needed to find cubes of the form $p^2 + 3q^2$ and Euler shows that, for any a, b if we put
$p = a^3 - 9ab^2$, $q = 3(a^2b - b^3)$ then
$p^2 + 3q^2 = (a^2 + 3b^2)^3$.
This is true but he then tries to show that, if $p^2 + 3q^2$ is a cube then an a and b exist such that p and q are as above. His method is imaginative, calculating with numbers of the form $a + b\sqrt{-3}$. However numbers of this form do not behave in the same way as the integers, which Euler did not seem to appreciate.
The next major step forward was due to Sophie Germain. A special case says that if n and $2n + 1$ are primes then $x^n + y^n = z^n$ implies that one of x, y, z is divisible by n. Hence Fermat's Last Theorem splits into two cases.
Case 1: None of x, y, z is divisible by n.
Case 2: One and only one of x, y, z is divisible by n.
Sophie Germain proved Case 1 of Fermat's Last Theorem for all n less than 100 and Legendre extended her methods to all numbers less than 197. At this stage Case 2 had not been proved for even $n = 5$ so it became clear that Case 2 was the one on which to concentrate. Now Case 2 for $n = 5$ itself splits into two. One of x, y, z is even and one is divisible by 5. Case 2(i) is when the number divisible by 5 is even; Case 2(ii) is when the even number and the one divisible by 5 are distinct.

Teksten findes på adressen:
http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Fermat's_last_theorem.html
History topic: Fermat's last theorem

Forløbet kan gennemføres i et samarbejde med engelsk med fokus på:

Andrew Wiles and his proof of Fermat's Last Theorem

Matematik bidrager med:

- Elementær talteori.
- Pythagoras' sætning.
- Pythagoræiske Tripler.
- Om beviser.
- Fermat og hans matematik.
- Eksempler på uløste matematiske problemer.

Litteratur: Steffen Due Bentzen: *Fermats Sidste Sætning – hvorfor er den stadig ikke bevist*, i Matematiske Ideer, Matematiklærerforeningen 1992.

Engelsk bidrager med:

- Tekster om mennesket og matematikken (*Men of Mathematics*, E.T. Bell, Div. Bøger om Wiles).
- Tekster om Pythagoras.
- Tekster om Fermat og hans tid (Simon Singh, *Fermat's Last Theorem*, Amir d. Aczel, *Fermat's Last Theorem: Unlocking the Secret of an Ancient Mathematical Problem*, Paulo Ribenboim, *Fermat's Last Theorem for Amateurs*).
- En video-udsendelse om Wiles, som er vist på DR.

Diverse web-materialer (f.eks.: http://www.pbs.org/wgbh/nova/proof/).